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Relaxation Dynamics of QDLs Subjected to Optical Feedback

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Abstract

Background: Quantum dot lasers (QDLs) are promising semiconductor laser devices due to their discrete energy levels and strong carrier confinement, which enable high-speed operation and improved performance in applications such as optical communications, sensing, and quantum information processing. The interaction of these lasers with optical feedback significantly affects their dynamic behavior and stability. **Objectives:** This study aims to investigate the relaxation dynamics and stability characteristics of quantum dot lasers subjected to optical feedback by analyzing the influence of injection rate and detuning frequency on the system behavior. **Methods:** A theoretical analysis based on a rate-equation model describing the complex electric field, carrier density in the wetting layer, and occupation probability in quantum dots was employed. The steady-state conditions were obtained, and the Jacobian matrix was used to derive an analytical expression for the growth rate. Numerical simulations were then performed to evaluate the relaxation oscillation frequency and damping rate and to compare the dynamics of QDLs with and without optical feedback. **Results:** The results show that optical feedback increases the relaxation oscillation frequency and decreases the damping rate as the injection rate increases, leading to reduced system stability and the emergence of instability regions. For negative detuning frequency, bistability regions appear, enabling potential applications in optical switching and optical data storage. **Conclusions:** Optical feedback significantly influences the dynamical response of QDLs, modifying their stability and oscillatory behavior. These findings provide insights for controlling laser dynamics and improving the design of quantum dot laser systems for advanced photonic applications.

Keywords: Quantum dot lasers, stability, relaxation oscillation, damping rates, rate -equation approach.

1. Introduction

In the realm of modern photonics, the development of novel semiconductor lasers has been a key point for researchers aiming to enhance device performance and functionality. Among them, Quantum dot lasers with their unique electronic structure and discrete energy levels have emerged as promising candidates for a wide range of applications [1], from optical communications [2] to quantum information processing [3]. One critical factor that significantly influences the dynamic response of these lasers is their interaction with optical feedback. Such optical feedback may be originated from external mirrors, optical source or unwanted reflections from the laser's operating environment. Understanding the interaction between quantum dot lasers and optical feedback has become important to meet the demands of advanced applications. Rate equations provide a fundamental framework for understanding the dynamics of quantum dot lasers (QDLs) by examining various factors. A lot of efforts have been directed toward formulating suitable rate equations for solitary QDLs [4,5,6,7,8]. Refining these rate equations, researchers can extend them to incorporate the effects of optical feedback, offering a more comprehensive view of the complex behaviours of QDLs [9,10,11,12].

The objective of this paper was to manipulate appropriate rate equations to study the dynamic response of QDLs under optical feedback, by finding an analytical expression of growth rate σ which in turn gives valuable insight about damping rate and relaxation oscillation frequency. In addition, a simulation method was used to validate the results by studying the response of QDL of a five-layer structure grown by solid-source MBE to optical feedback.

The rate equations for a QDLs subject to optical feedback were formulated by D. Goulding [14], which include three equations for the complex electric field E, the occupation probability in a dot ρ, and the carrier density n in the wetting layers. The complete set of equations are:

$$E' = \frac{1}{2}(1+i\alpha)[-1+g(2\rho-1)]E + \Gamma e^{i\Delta t} \qquad (1.1)$$

$$\rho' = \eta[Bn(1-\rho) - \rho - (2\rho-1)E^2] \qquad (1.2)$$

$$n' = \eta[J - n - 2Bn(1-\rho)] \qquad (1.3)$$

where the prime means differentiation with respect to $t = \frac{t'}{\tau_{ph}}$. α is the linewidth enhancement factor. Γ : injection rate. Δ : detuning frequency. $B = \tau\tau_{cap}^{-1}$ Dimensionless capture rate, where τ is the carrier recombination time and τ_{cap} is the capture time. $\eta = \tau_{ph}\tau^{-1}$ is the ration between photon lifetime τ_{ph} and the recombination time τ.the factor $1-\rho$ is the Pauli blocking factor. The factor 2 in eq 1.3 accounts for the spin degeneracy in the QD energy level.

The previous rate equations were analyzed. This involved examining the effect of small perturbations around the steady state condition by taking derivatives of the rate equations and setting them equal to zero to find the steady state condition. After that, the partial derivative with respect to the state variables were used to construct the jacobian matrix. The characteristic equation was applied to evaluate the eigenvalues of this matrix at the steady state. These eigenvalues would provide valuable insight about relaxation oscillation and damping rate.

2. Reduced Rate Equations

The researchers first reduced the rate equation by introducing $\epsilon = g-1$, $\rho = 1+\epsilon u$, $s = \epsilon t$, $\gamma = \epsilon^{-1}\Gamma$, $\delta = \epsilon^{-1}\Delta$

Then the following equation was obtained:

$$E' = \frac{1}{2}(1+i\alpha)[2u(1+\epsilon)]E + \gamma e^{i\epsilon\delta\epsilon^{-1}s} \qquad (2.1)$$

$$u' = \eta\epsilon^{-2}[-Bnu\epsilon - 1 - \epsilon u - (2\epsilon u + 1)E^2] \qquad (2.2)$$

$$n' = \eta\epsilon^{-1}[J - n + 2Bn\epsilon u] \qquad (2.3)$$

the prime means differentiation with respect to s. Since $\epsilon^{-2} \gg \epsilon^{-1}$ $as\ \epsilon \to 0$, u is faster than n and it could be eliminated adiabatically from equation (2.2) then

Also, the complex electric field was composed of both the real part, which is related to the amplitude, and the imaginary part which is related to the phase. It could be substituted as:

$$E = R\, e^{i\delta s + i\varphi} \qquad (2.4)$$

Then the reduced rate equations were:

$$R' = \frac{1}{2}\left[1 - \frac{2(1+R^2)}{Bn\epsilon}\right]R + \gamma cos\varphi \qquad (2.5)$$

$$\varphi' = -\delta + \frac{1}{2}(\alpha)\left[1 - \frac{2(1+R^2)}{Bn\epsilon}\right] - \frac{\gamma sin\varphi}{R} \qquad (2.6)$$

$$n' = \eta\epsilon^{-1}[J - n - 2(1+R^2)] \qquad (2.7)$$

Understanding the impact of these parameters on the dynamical properties of a QD laser was precisely the objective of our work.

3. Growth Rate

From equations 2.5 to 2.7, the researchers determined the steady states and conditions. For simplicity, they introduced:

$$F = \frac{1}{2}\left[1 - \frac{2(1+R_s^2)}{B\epsilon(J - 2(1+R_s^2)}\right] \qquad (3.1)$$

Then, the steady state conditions were:

$$FR_s = (\gamma cos\varphi)_s \qquad (3.2)$$

$$(-\delta + \alpha F)R_s = (\gamma sin\varphi)_s \qquad (3.3)$$

$$n_s = J - 2(1+R^2) \qquad (3.4)$$

By taking the square of both sides of (3.2) and (3.3) and adding them the researchers found:

$$(F^2 + (-\delta + \alpha F)^2)R_s^2 = \gamma^2 \qquad (3.5)$$

From the linearized equations, the researchers then formulated the characteristic equation for the growth rate [16]: σ

$$\sigma^3 + a_1\sigma^2 + a_2\sigma + a_3 = 0 \qquad (3.6)$$

Where:

$$a_1 = -(G - F - \eta\epsilon^{-1}) \qquad (3.7)$$

$$\begin{aligned} a_2 &= -(FG + G\eta\epsilon^{-1} - F\eta\epsilon^{-1}) \\ &+ \left((\delta - \alpha F)\alpha(F - G) + (\delta - \alpha F)^2\right) \\ &+ (1 - 2F)(F - G)B\eta \end{aligned} \qquad (3.8)$$

The researchers could expand in powers of since it did not appear in steady state expressions [13], the researchers introduced   in jacobian matrix, then they equated each coefficient of to zero, which led to set of equation in term of and . Solving these equations, the researchers obtained:$\sigma\eta\sigma = \eta^{\frac{1}{2}}\lambda_0 + \eta\lambda_1 + \cdots \eta\ \lambda_0\lambda_1$

$$\sigma_1 \approx \pm i\sqrt{(F-G)\left(\frac{2}{5}\sqrt{\begin{matrix}2\dfrac{(1-2F)B-2\epsilon^{-1}}{5} \pm \\ \begin{matrix}((1-2F)B-2\epsilon^{-1})^2 \\ -5\epsilon^{-1}(\epsilon^{-1}+(1-2F)B)\end{matrix} \\ +\epsilon^{-1}+(1-2F)B\end{matrix}}\right)\eta}$$

$$+(\frac{(1-2F)B-2\epsilon^{-1}}{5}$$

$$\pm\frac{1}{5}\sqrt{\begin{matrix}((1-2F)B-2\epsilon^{-1})^2 \\ -5\epsilon^{-1}(\epsilon^{-1}+(1-2F)B)\end{matrix}})\eta \qquad (3.10)$$

the real part represents the damping rate which is related to the stability of the mode while the imaginary part represents the RO frequency of the mode. When the real part is negative, the mode exhibits stability and the excited oscillations damped out for a time development with frequency determined from the imaginary part. However, a positive real part means the unstability of the mode and the laser exhibits regular or irregular oscillations with a frequency corresponding to the imaginary part [15].

Table 1. Physical parameters used in the simulation of the QDL model unless stated otherwise.

Parameter	Value
g	1.01
ϵ	0.01

B	10^2
J_{th}	4
α	1.2
η	2×10^{-3}

4. Numerical Results

In order to investigate the previous results, the researchers examined the response of QDL of a five-layer structure grown by solid-source MBE to optical feedback. This QDL consist of 2.4 InAs monolayers topped with 5 nm GaInAs stacked in a 400 nm thick optical cavity. A GaAs spacer of 35 nm is also used between the QD layers. The constants shown in table 1 below are used in the following discussion [17].

The researchers characterized the dynamics of QDL subjected to the optical feedback that operate above threshold region $J => J_{th}$. The control parameters that are varied are the detuning frequency and the injection rate.

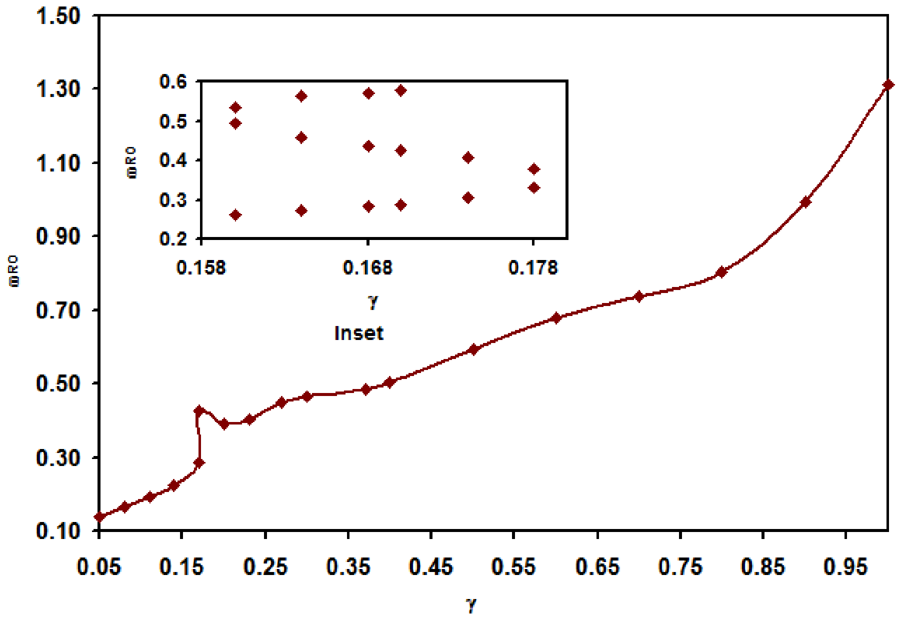

Figure 1. RO frequency ω_{RO} versus injection rate γ for $\delta = -0.4$. The inset represents different value of RO frequency ω_{RO} in bistability region.

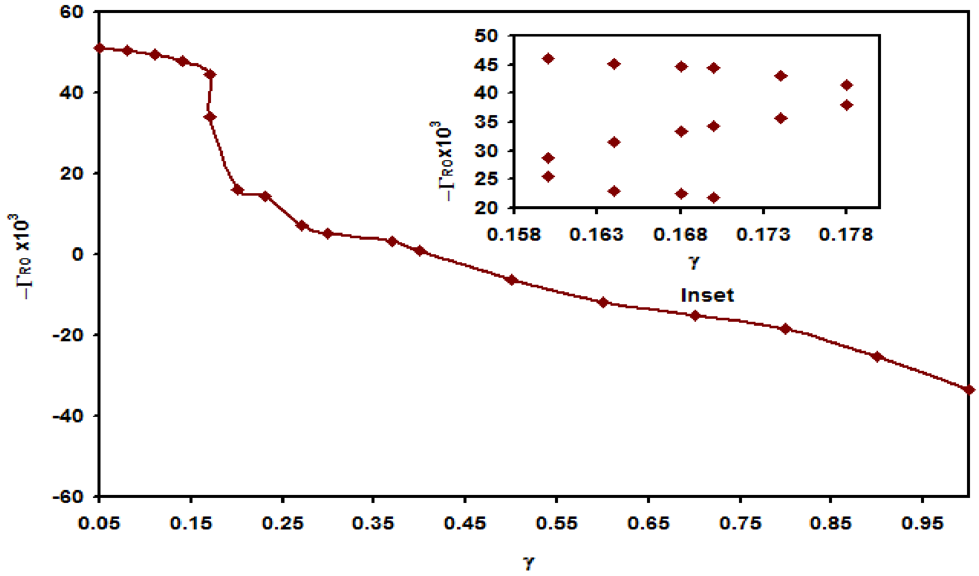

Figure 2. Damping rate Γ_{RO}versus injection rate γ for $\delta = -0.4$. The inset represents different value of damping rate Γ_{RO} in bistability region.

To visualize the effect of changing the injection rate in the dynamics of the system, the researchers manipulated two cases: one for negative and other for positive (=) detuning frequency by varying the injection rate up to. For each negative and positive detuning frequencies, the RO frequency increased and the damping rate decreased as illustrated in figures 1, 2 and 3. This implied that further increasing of the injection rate, the system tends toward instability and chaotic behaviour. In both cases, the solution gave a negative damping rate for low injection rate, which implied the stability of the system (Yamada, 2014). Further increasing of injection rate, the damping rate becomes positive and so the system goes to instabilities. It is worth noting that for (=) the system exhibts stability over a wide range of injection rate than . In the case of , different values of both damping rate and relaxation oscillation for the same injection rate in the inset of figure 1 and 2 indicated the appearance of bistabilty or excitability. While it is absent for Bistabillity means the ability of the system to operate at two stable states with different intensities [18], which enables the system to transit from one state to another at specific conditions. This provides the opportunities for control and manipulation the laser output which can be utilized in various optical technologies such as optical switching [19] and optical memory. $(\delta = -0.4)$ δ 0.4 (1)δ 0.4$(\delta = -0.4)(\delta = -0.4)(\delta = 0.4)$.

At constant injection rate $(\gamma = 0.2)$, the RO frequency and the damping rate were plotted versus the detuning frequency as shown in figures 4 (a) and (b), respectively. Increasing the detuning frequency at constant injection rate had a noticeable effect on the RO frequency and the damping rate. Specifically, as the detuning frequency increased, the RO frequency decreased, and the damping rate increased. This pattern indicated an improvement in the stability of the system. In this context, the detuning frequency is defined as the difference between the master laser and the slave laser frequencies. In our scenario, this detuning frequency δ corresponds to the difference between the frequency of the laser reflected back into the cavity ω_L and the resonance frequency of the cavityω_0. It's worth noting that the resonance frequency depends upon the cavity length. Consequently, controlling the detuning can be achieved by adjusting the cavity length. Therefore, enhancing stabilization can be accomplished through precision manufacturing processes.

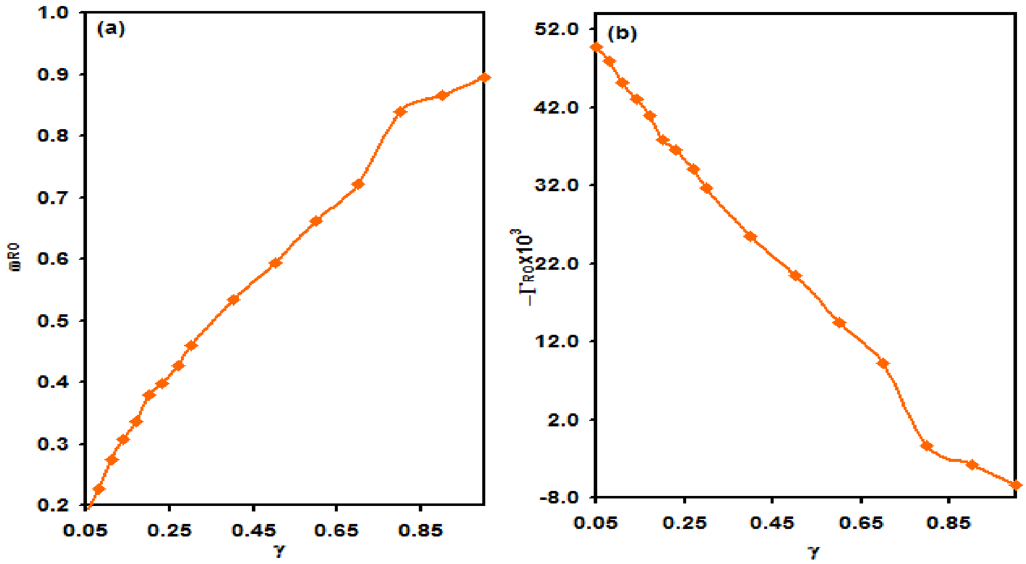

Figure 3. (a) RO frequency ω_{RO} versus the injection rate γ for $\delta = 0.4$ and (b) Damping rate Γ_{RO} versus injection rate γ for $\delta = 0.4$.

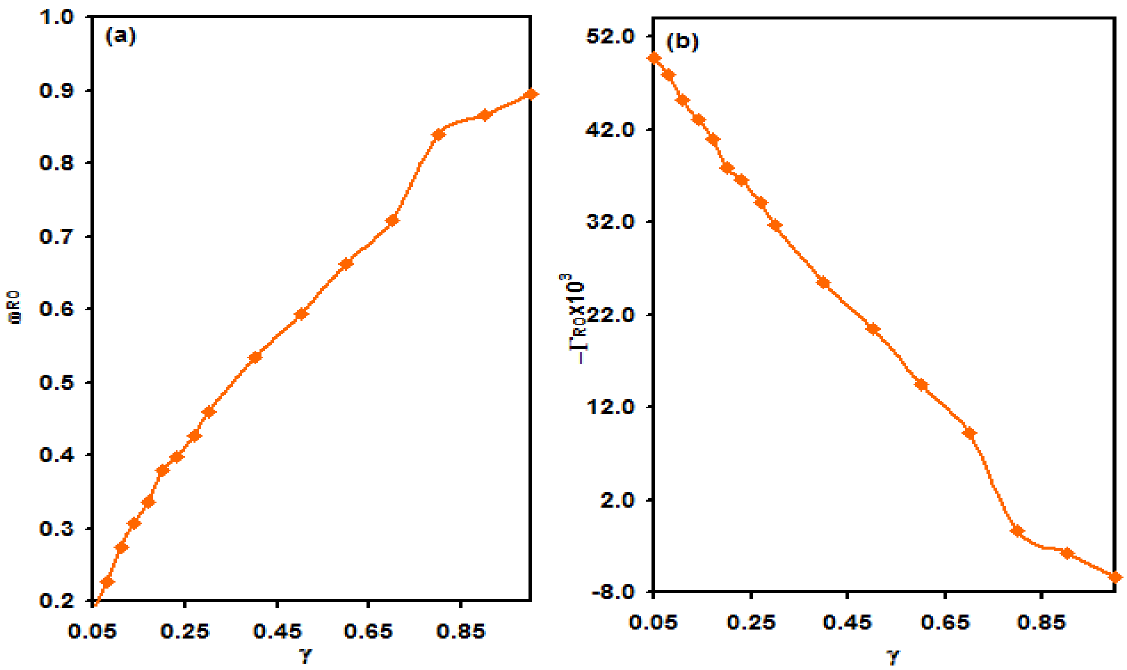

Figure 4. (a) RO frequency ω_{RO} versus the detuning frequency δ at constant injection rate γ.And (b) Damping rate Γ_{RO} versus the detuning frequency δ at constant injection rate γ.

5. Numerical Comparison between Dynamics of solitary and subjected to optical feedback QDLs.

In this section, the researchers aimed to compare the impact of optical feedback on the dynamics of QDLs for one case where ($\delta = -0.4$) with those of solitary QDLs. It is well known that the RO frequency in the presence of optical feedbacks shifts from that of solitary oscillation (Ohtsubo, 2013). However, in our case it took larger value than that of the solitary oscillation at the same value of steady state intensity as shown in figures 5(a).

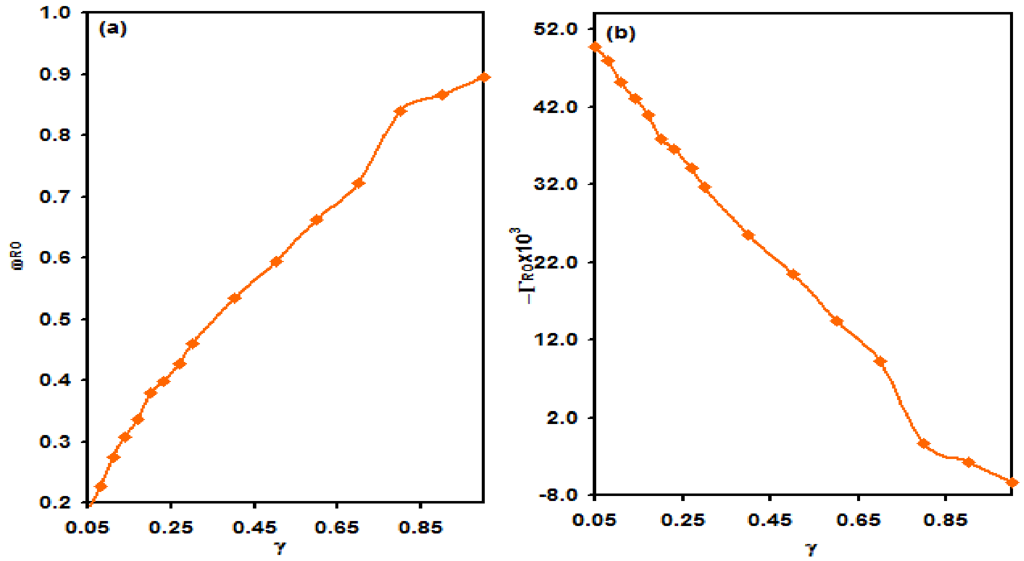

Figure 5. (a) RO frequency ω_{RO}versus steady state intensity I_s for solitary and under optical feedback QDL and (b) Damping rate Γ_{RO} versuse steady state intensity I_s for solitary and under optical feedback QDL.

In a stability regime, the damping rate of the oscillations was lower than the damping in the solitary oscillation which indicated that the system is less stable than predicted from solitary model. Moreover, there was an instability regime indicated by positive damping rate. Furthermore, as explained previously, the optical feedback provides bistability phenomena, which provide an ability to use as optical switch.

6. Conclusions

In this work, the dynamic response of QDLs subjected to optical feedback was explored analytically utilizing a rate equation model developed by O'Brien et al. An expression for the growth rate was derived analytically in terms of the eigenvalues of the Jacobian matrix evaluated at steady state. Our findings were numerically validated by studying the reaction of a five-layer structure generated by solid-source MBE to optical feedback. The computation revealed that under optical feedback at constant detuning frequency the RO frequency increased while the damping rate decreased as the injection rate increased. For small injection rate, the system was still stable. However, with further increasing of the injection rate the system went to instabilities and chaotic region. Moreover, for negative detuning frequency, there was a bistability region which enables to use the laser in various applications such as optical switch and optical data storing. Comparing to solitary QDLs, the analysis revealed that the system is less stable than predicted from solitary model.

8. Declarations

Acknowledgments

Not applicable.

Ethical consideration

Not applicable.

Consent to participate

Not applicable

Conflicts of interest

Not applicable

Data availability

Declared upon request

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Author(s) Contribution

Conceptualization (C): M Abusaa and AA Saleh. **Methodology (M):** Haneen NM Jaradat, M Abusaa and AA Saleh. **Formal Analysis (FA):** Haneen NM Jaradat. **Writing – Original Draft (WO):** Haneen NM Jaradat. **Writing – Review & Editing (WR):** Haneen NM Jaradat, M Abusaa and AA Saleh. **Visualization (Vi):** Haneen NM Jaradat **Supervision (Su):** M Abusaa and AA Saleh.

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